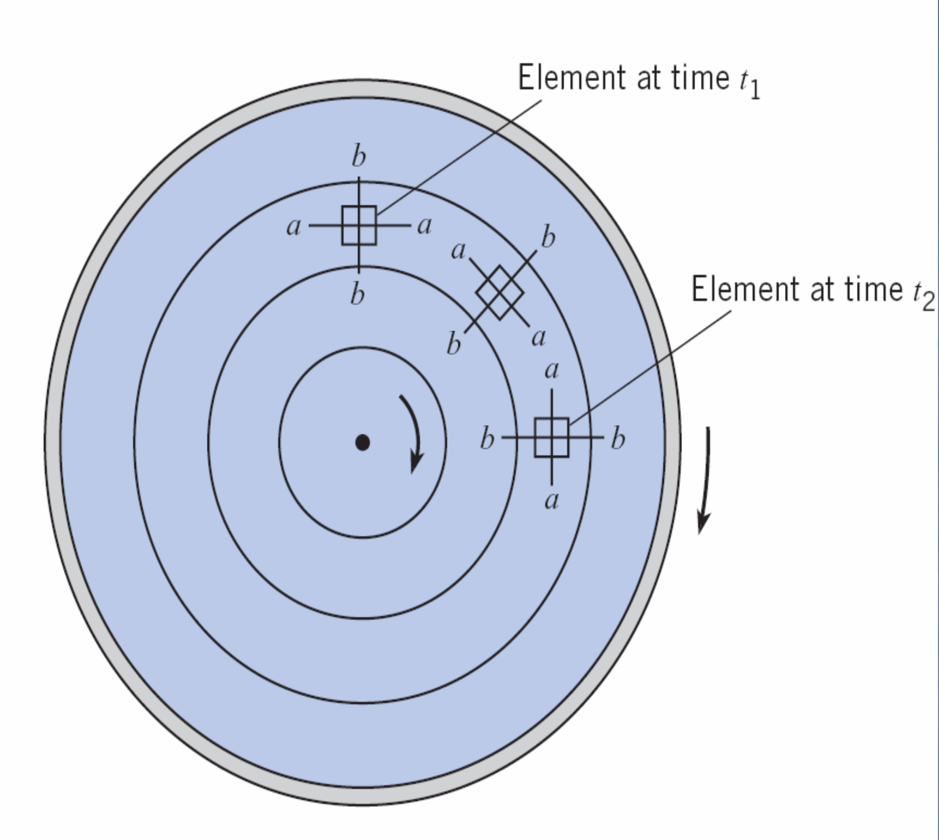
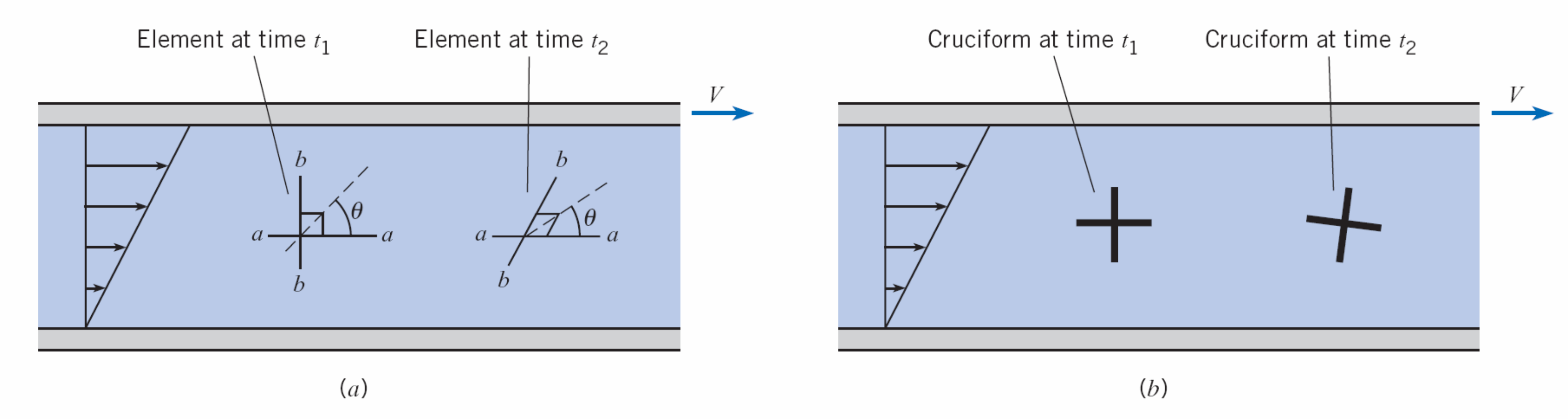
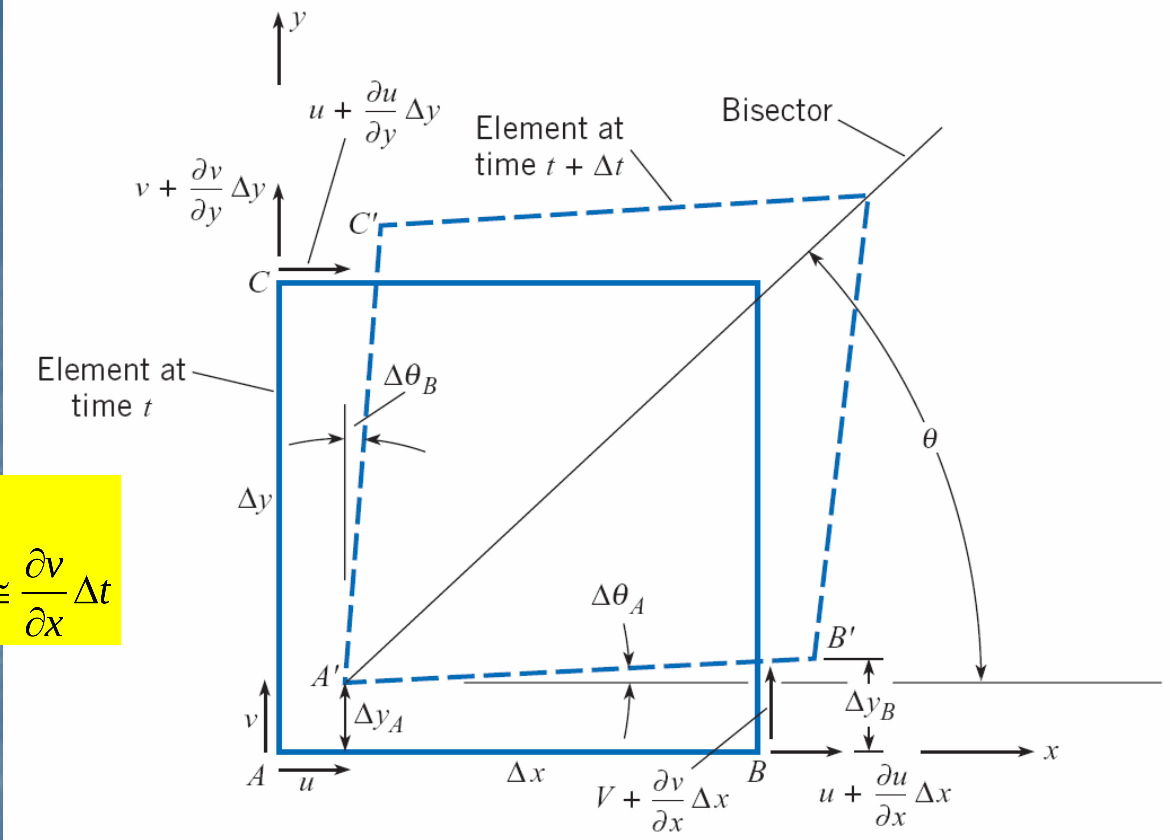
Rotation


Element at time t_1
Element at time t_2
b
a
a
b
a
b
b
a
a
b
b
a


Element at time t_1
Element at time t_2
Cruciform at time t_1
Cruciform at time t_2
V
V
b
a
a
b
θ
b
a
a
b
θ
(a)
(b)

The rotational rate is given as

$$\dot{\theta} = \frac{1}{2}\left(\dot{\theta}_A - \dot{\theta}_B\right)$$



$$\Delta\theta_A \cong \frac{\Delta y_B - \Delta y_A}{\Delta x} \cong \frac{\left(v + \frac{\partial v}{\partial x}\Delta x\right)\Delta t - v\Delta t}{\Delta x} \cong \frac{\partial v}{\partial x}\Delta t$$

$$\Delta t \to 0$$

$$\dot{\theta}_A = \frac{\partial v}{\partial x} \qquad \dot{\theta}_B = \frac{\partial u}{\partial y}$$

So the rotation rate of the element about the **z-axis (normal to the page)**

$$\dot{\theta} = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad \Omega_z = \frac{1}{2}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)$$

Ω **is the rotational velocity**

Fluid Motion

The rotational velocity about other axes

$$\Omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$$\Omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

$$\Omega_z = \frac{1}{2}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)$$

An irrotational flow $(i.e.\ \Omega = 0)$ **requires**

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$

$$\frac{\partial w}{\partial y} = \frac{\partial v}{\partial z}$$

$$\frac{\partial u}{\partial z} = \frac{\partial w}{\partial x}$$

Vorticity $(\omega) = 2\Omega$

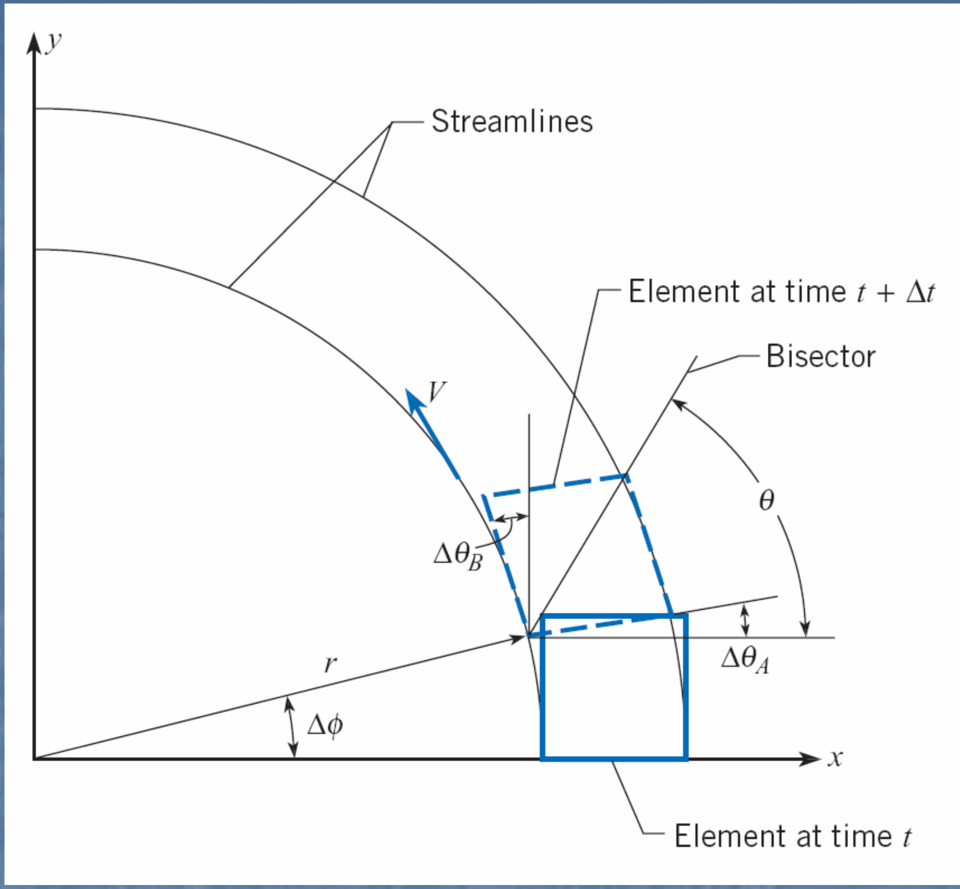
Rotation in flows with Concentric Streamlines

Consider the Fig. aside, the rotation of the element is quantified by the Rate rotation of the bisector, which is

$$\dot{\theta} = \frac{1}{2}\left(\dot{\theta}_A + \dot{\theta}_B\right)$$

But $\Delta\phi = \Delta\theta_B$ from geometry

And $\dot{\phi} = \frac{V}{r} = \dot{\theta}_B$ $\quad \dot{\theta}_A = \frac{\partial V}{\partial r}$



$$\dot{\theta} = \Omega_z = \frac{1}{2}\left(\frac{dV}{dr} + \frac{V}{r}\right)$$

$$V = \omega r$$

$$\dot{\theta} = \Omega_z = \frac{1}{2}\left(\frac{d(\omega r)}{dr} + \frac{V}{r}\right)$$

$$\dot{\theta} = \Omega_z = \frac{1}{2}\left(\frac{d(\omega r)}{dr} + \omega\right)$$

i.e. $\Omega_z = \omega$

Fluid Motion

$\Omega_z = \omega$ **For rotational flow Or Forced Vortex**

For irrotational flow $\Omega_z = 0$

$\left(\frac{dV}{dr} = \frac{V}{r}\right)$ **or** $\left(\frac{dV}{V} = \frac{dr}{r}\right)$

$V = \frac{C}{r}$ **or** $Vr = C$ **Free Vortex Flow**

In Free Vortex Flow, the Tangential Velocity Varies Inversely with Radius

END OF LECTURE (5)